Reply:

Our proposed method RNDDR[1] can be interpreted using either an extended Kalman filter (EKF) perspective or a moving horizon estimator (MHE) perspective. In our development of RNDDR we have presented RNDDR as a modification of the extended Kalman filter (EKF) to handle constraints in nonlinear state estimation. There are several useful inferences that we can draw by interpreting RNDDR in this manner, which are not easy to derive using an MHE viewpoint. The two main problems with EKF are (a) its inability to handle state and other types of constraints, and (b) the inaccuracy introduced due to the linearization of the nonlinear model for estimating the error covariance matrix. Our viewpoint is that these problems can be resolved using distinct solution strategies. The central message in our approach is that EKF can be modified to effectively handle state constraints, and it is not necessary to use a general purpose MHE to tackle this problem. Subsequent to our articles, Ungarala reaches the same conclusion in his article presented in an AIChE conference,[2] and in a article later published in another conference (see the conclusions section of Ungarala et al.[3]). In his work,[2,3] he derives an estimator termed constrained extended Kalman filter (CEKF), which he states is equivalent to MHE-1. Surprisingly, the conclusion that he arrives at in those publications are completely contrary to the arguments that he makes against MHE with horizon of one (MHE-1) in his letter to the editor.

Robertson et al.[4] have indeed provided an excellent discussion on the equivalence between MHE and EKF. Based on the article by Robertson et al.,[4] one could also view RNDDR as MHE-1. The equivalence between RNDDR and MHE-1 can be established only if the prior estimate and its error covariance matrix are computed in exactly the same manner in MHE-1 as is done in RNDDR. It is to be noted that in RNDDR, the constrained state estimate obtained at each sampling instant is used to obtain the predicted state estimate and to linearize the nonlinear model. The predicted estimate error covariance is obtained using the resulting linearized system matrix. Robertson et al.[4] have suggested several possible methods for computing the prior state estimate and its error covariance (also referred to as arrival cost). However, the implementation details are crucial. In fact, the choice considered as MHE-1 by Ungarala in his note is not the RNDDR approach, and this choice will inherit all the problems of EKF. Further, the choice outlined by Ungarala would make that particular version of MHE-1 nonrecursive, whereas RNDDR is recursive.

The importance of the EKF perspective is reflected by the fact that articles are still being published[5] for solving equality constrained state estimation problems in linear systems. In Teixeira et al.[5] published in 2007, an equality constrained EKF (CEKF) approach is discussed. One could argue that this approach is MHE-1 with Kalman filter used for approximating the arrival cost.

We strongly believe that viewing every algorithm as a MHE with a corresponding nonlinear filter for approximating arrival cost does not always provide a useful perspective for making progress in this area. In some cases, an alternate EKF viewpoint allows one to develop other forms of suboptimal filters for nonlinear problems. We have developed a version of a particle filter, the URNDDR approach,[6] starting from the RNDDR viewpoint. In this approach, unscented transformation is used in combination with the RNDDR optimization. We have also developed another suboptimal filter that corrects the UKF estimate using a single step RNDDR type optimization,[7] thereby improving the computational properties of the URNDDR approach. These advancements have been possible by viewing RNDDR as an extension to EKF.

We also reject the off-hand comment by Ungarala about RNDDR performance using a simulation example from the URNDDR[6] article. Ungarala has misinterpreted the batch process result discussed in our URNDDR[6] article. We clearly mention in section 5 of the URNDDR[6] article that the tuning parameters have been deliberately chosen to show that EKF and UKF will not converge to the true dynamics when constraints are present. Further, for a similar simulation run, it was shown that a clipping strategy implemented on a EKF results in predicted pressures 3-orders of magnitude larger than the measured pressure (Figure 3 in Haseltine and Rawlings[8]). The RNDDR results are orders of magnitude better compared to that result. The result for RNDDR (Figure 2 of Vachhani et al.[6]) is with the same poor initial guess for the prior as the UKF result (Figure 1 of Vachhani et al.[6]). Furthermore, the URNNDR result (Figure 3 of Vachhani et al.[6]) is also for the same poor guess. As clearly mentioned in that article, we showed this as a classic example which demonstrates that for the sole purpose of achieving reasonable convergence of the estimator, the imposition of constraints is sufficient as done in RNDDR, while the quality of the convergence (performance) can be improved by improving the accuracy of covariance estimation as additionally carried through unscented transformation in URNDDR. In fact, Haseltine and Rawlings[8] mention that changing arrival cost approximations when the constraints are active as a way of addressing the accuracy of estimation without having to increase the window size.

In our RNDDR[1] article, we had shown simulation studies of several processes with constraints (CSTR, polymerization reactor) where RNDDR performs satisfactorily for state and parameter estimation with the correct tuning parameters. We, in fact, presented several case studies to counter the general argument in the literature that one needs large horizons for constrained state estimation. Proper tuning of RNDDR is sufficient in most cases. We do not see any evidence of poor performance in any of the case studies that we have shown in the RNDDR[1] article itself.

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